

# A TransDimension Visualization Model for Complex Dynamic System Visualizations

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***Abstract**—Linear and non-linear controllable systems are commonly found in many engineering problems and are examples of models that involve complex high-dimensional spatially-related data sets. Large-scale systems comprise many state variables that evolve in complex ways. Particular evolution trajectories and regionalized evolutionary behavior of such complex systems are often non-intuitive and may reflect bifurcations in the system. A visualization model that includes multiple imagery and animation elements that facilitate heightened understanding of such a system's evolutionary aspects is proposed. The visualizations are based on information represented in a novel structure termed Orthogonal Organized Finite State Machines (OOFSM); a brief review of an OOFSM is included. The goals of the visualization model are to support the analysis of the complex dynamic system by analysts seeking to understand the operation and behavior of the system and at the same time to support the decision making process inherent in controlling the physical system.*

Linear and non-linear controllable systems are commonly found in many engineering problems and are examples of models that involve complex high-dimensional spatially-related data sets. Such systems range in size and complexity from small-scale systems of a few state variables to large-scale systems comprising many state variables that evolve in complex ways. Particular evolution trajectories and regionalized evolutionary behavior of such complex systems are often non-intuitive and may reflect bifurcations in the system. For large-scale systems in particular, the state space exists in high dimensional spaces and many of the transitions are expected to be intra-dimensional. Understanding the operation and behavior of the system together with the details about specific evolution trajectories and how such relates with other trajectories or with the system as a whole may provide insight into the underlying dynamics of the system as it evolves

towards desired or undesired behavior. In addition, this may also facilitate decision making by policy makers who are involved in control operations.

Typical visualizations of the state space are two or three dimensional plots (see for example [1], [2]), often generated by specialized software such as MATLAB, that illustrate the projection of trajectories or vector fields upon the selected dimensions. Animations of the trajectory path, sometimes combined with physical system simulation, provide additional information about the time progression of the system's evolution. State space plots also provide graphical displays of aspects of the system. However, limitations of these approaches include the difficulty in visualizing the system's regional behaviors and associated properties especially in a reduced geometric space such as the typical two and three dimension plots. Moreover, the state space may be neither perceptively nor intuitively correlated. An even larger problem is to provide a context that enables the understanding of an overview perspective combined with details of system's behavior. Other work in the area includes [3].

The visualization model proposed in this paper is based on immersive navigation in and around novel structures termed Orthogonal Organized Finite State Machines (OOFSM). An OOFSM represents a lattice partitioned, and therefore a discretized, state space of a dynamic system. We emphasize the properties associated with trajectories of the system in the context of regions of behavior around the trajectories. The visualization approach includes multiple imagery and animation elements that facilitate heightened understanding of the system's evolutionary aspects (see [4]). This approach also

enables query-like capability such as: What bifurcations exist in the normal operation or in the event of a cascading failure of the dynamic system?

The rest of this paper is organized as follows. The OOFSM is briefly reviewed in the next Section, Section I. The visualization model together with its demonstration are presented in Section II. Conclusions are given in Section III.

## I. REVIEW OF ORTHOGONAL ORGANIZED FINITE STATE MACHINES (OOFSM)

Orthogonal Organized Finite State Machines (OOFSM) [5] represent a lattice partitioned, and therefore a discretized, state space of a dynamic system. Formally, it is defined by  $M = (Y, \mathcal{L}, \bar{V}_Y)$ . A lattice partitioning  $\mathcal{L}$  applied to an  $n$  dimension state space  $X = \{x_1, x_2, \dots, x_n\}, x_i \in \mathbb{R}$  leads to a set of discretized states  $\mathcal{L} : X \rightarrow Y$  where  $Y = \{y_0, y_1, \dots, y_{o-1}\}$  for some finite  $o$ . In general,  $\mathcal{L}$  defines a set of partition boundaries  $P = \{p_{i,j} | 1 \leq i \leq n, 0 \leq j \leq o-1\}$  with  $p_{i,j} \in \mathbb{R}^{n-1} = (b_l, b_u)_{i,j}, b_u > b_l$ . Each  $p_{i,j}$  is aligned normal with the corresponding  $i$ th state variable;  $\iota(b)$  denotes this value. A discrete direction vector field  $\bar{v}_j = (\dots, a_i, \dots)$  where  $a_i = \bigcup_k v_{j k_i}$  is the union of a set of discrete direction vectors  $\{v_{j k} | k \geq 1\}$  in state  $y_j$  of  $Y$ ;  $a_i \in \{-1, 0, 1\}$ . The intersection of a trajectory  $e \in E$  with  $p_j \in P$  for a fixed  $j$  derives  $v_j$ ; the intersections of all  $e \in E$  with  $p_j \in P$  derives  $\bar{v}_j$  for a fixed  $j$ . Lastly, the set  $\bar{V}_R = \{\bar{v}_j | j \in R\}$  for region  $R$  defines a region field;  $\bar{V}_Y$  denotes some general region field. A uniform region field has the same region field for each  $y_j \in R$ . For convenience, elements of  $Y$  may be interchangeably expressed in terms of the dimension of the system. Figure 1 illustrates an OOFSM for:  $n = 2$ , uniform unit  $\mathcal{L}$  so that  $o = 16$  and  $P = \{p_{10}, p_{20}, p_{11}, p_{21}, \dots, p_{1j}, p_{2j}, \dots, p_{115}, p_{215}\}$  such that  $p_{10} = (b_{l_1(0,0)}, b_{u_1(0,0)})$  where  $\iota(b_{l_1(0,0)}) = 0$  and  $\iota(b_{u_1(0,0)}) = 1$  (i.e., the values on the  $x_1$  axis corresponding with the lower and upper boundaries of the ‘vertical’ partition pair comprising the ‘left’ and ‘right’ sides of state  $y_{0,0}$ ) and so forth with  $X = \{x_1, x_2\}$ , and  $\bar{V}_Y = \bar{V}_{R_1} \cup \bar{V}_{R_2}$  where  $\bar{v}_{0,3} = (\{0\}, \{0\})$  defines the uniform region field  $\bar{V}_{R_1}$  for  $R_1 = \{y_{k,3} | 0 \leq k \leq 3\}$  and  $\bar{v}_{0,0} = (\{0\}, \{1\})$  defines the uniform region field  $\bar{V}_{R_2}$  for  $R_2 = \{y_{k,l} | 0 \leq k \leq 3, 0 \leq l \leq 2\}$  (i.e., there are

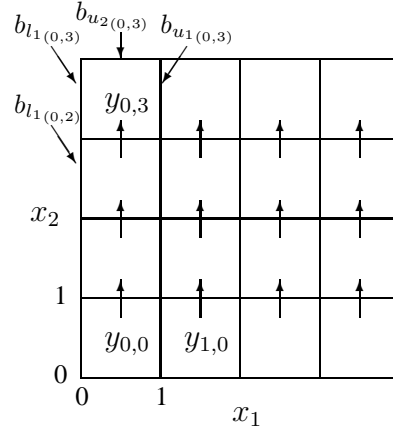


Fig. 1. Illustration of state space definitions,  $n = 2$ ,  $o = 16$  and uniform unit partition  $\mathcal{L}$ .

two uniform region fields with the first being null (terminal states) associated with the ‘top row’ and the second being ‘upwards only’ associated with the remaining states).

## II. VISUALIZATION MODEL

The two broad goals of the visualization model are:

- to support the analysis of the complex dynamic system by analysts seeking to understand the operation and behavior of the system,
- to support the decision making process inherent in controlling the physical system.

We propose a visualization model that is perceptively correlated with the state space. Both immersive and non-immersive interactions are supported. Other key elements include color and animation. A consistent context is maintained where the user identifies with one or more trajectories and/or one or more regions in the state space. The context also provides for additional specific related information to be embedded into the visualization, including, support of sensor related information including geographic placement and correlation of sensor related information with a trajectory. Combined, the visualization model primarily facilitates the perceptive and analytic understanding of state space transitions and dynamics, especially, as the system evolves towards desired or undesired states.

This section presents the visualization model by firstly, describing the key elements of the metaphor,

and secondly, by discussing the mapping of useful information to the visual primitives.

### A. Visualization Metaphor

A TransDimension  $n$  Domain model (TD <sup>$n$</sup> D) is proposed for the visualization of high dimensional state space representations of dynamic systems. The TD <sup>$n$</sup> D is composed of the following high-level components.

- **TransDimensional Corridor:** This is the primary visualization primitive and provides a common frame-of-reference throughout the visualization environment perceptively supporting the concept of strolling through the state space. The corridor has a defined opening portal where the user initiates a stroll. The corridor has four surfaces corresponding to up, down, left and right. Visual primitives may be applied to these surfaces to reflect the behaviors of the state space region being traversed, the physical and tangible events correlated with these regions or other metrics about the path of transitions, etc. The corridor supports at rest, forward and backward motion as well as three-axis rotation of the viewpoint (the virtual location of the user).
- **TransDimensional Window:** This provides a representation for hyper-plane cuts of the state space and visually appears as a window on one corridors surfaces. As a glyph, the window provides visual attributes such as window's length and height, the window frame's shape, color and texture as well as the window's relative position on the surface. The window's pane provides 'sight' into the state space 'beyond' the window. The terms 'sight' and 'beyond' are perceptible to the user; semantically, the data set corresponding to the state space is filtered or otherwise manipulated to provide meaningful information to the user. Filtration and manipulation is controlled by a control panel associated and with and located by each window. Static windows are specific to each state. Such windows are most useful to display specific information about each state. Traversal windows follow the viewer as the viewer strolls along the corridor. Animation can be employed

to change the window's display from state to state.

- **TransDimensional Doors:** This provides a representation for hyper-relocation through the state space. Hyper-relocation dissolves the current corridor in favor of a new corridor elsewhere in the space. The conditions for relocation are dependent upon a purpose of navigation and exploration, but not on the initiating of a new stroll. A history of relocations is maintained and is available by a pop-up control panel appearing in front of the user. The user is easily able to relocate back to any of the previous corridors.
- **Control Center:** This provides the primary non-immersive primitive useful to provide a comprehensive overview over all active corridors including those hyper-located. Either a toggle or dual display provides user activation. At system start-up, a blank control center is provided to the user. Doors are used to initiate a corridor. Windows are also enabled so as to present information in a consistent context.

### B. Visual Primitive Mapping

This section outlines the proposed mappings between the data domain as well as associated data sets and the visual primitives that make up the visualization.

- The corridor metaphor provides the context for a path of transitions, Figure 2. The path  $p$  is displayed along the linear extent of the corridor:  $(y_0, y_1, \dots, y_i \dots, y_N)$ . The corridor is visually sectioned off into segments representing specific states. A state transition path is expected to assume a twisted shape since many of the transitions in the system should be between dimensions. However, one aspect of interest is to facilitate the understanding and prediction of state transition paths that lead to regions of failure. To provide for this, the corridor is 'linearized' to provide a straight line-of-sight viewing of information 'down' the corridor. This enables the viewer to 'see' the set of states potentially far 'down' the path. To further facilitate this, each segment is appropriately highlighted according to the status of the system's

operation. Figure 3 shows an example trajectory of a path  $p = \{(0, 0, 0), (1, 0, 0), (1, 1, 0), (2, 1, 0), (2, 1, 1), (2, 2, 1), (2, 2, 2)\}$ ,  $|p| = 7$  and therefore the corridor has seven segments with the starting portal located at the entrance to  $(0, 0, 0)$ . The concept of path also invokes the concept of time. Time is correlated with the forward motion through the corridor. However, since each segment represents a state and since there may be a variable number of state transitions per unit time, time is not constant with respect to each state transition.

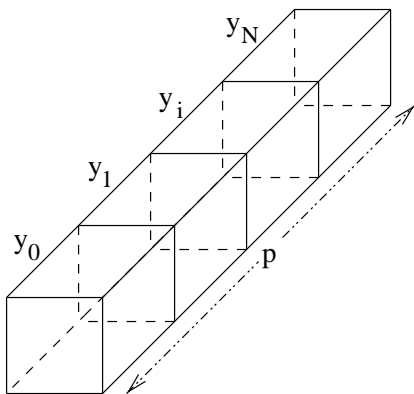


Fig. 2. A linearized corridor of discrete states

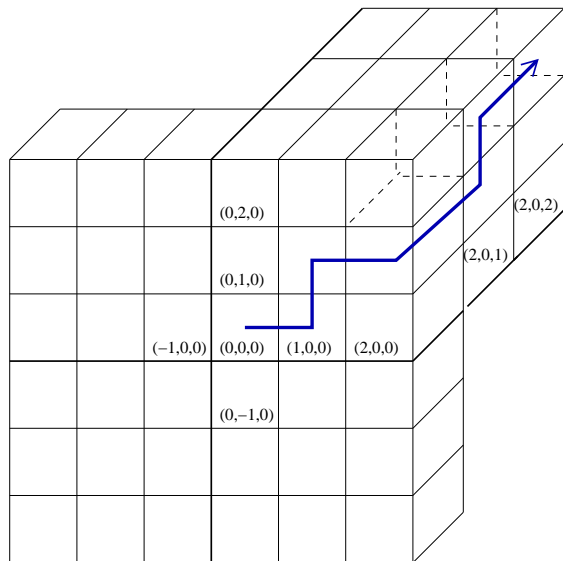


Fig. 3. An  $n = 3$  OOFSM illustrating a path (discrete trajectory)

- The window motif provides capability to embed multiply linked information about the domain and data sets. Windows exist within the

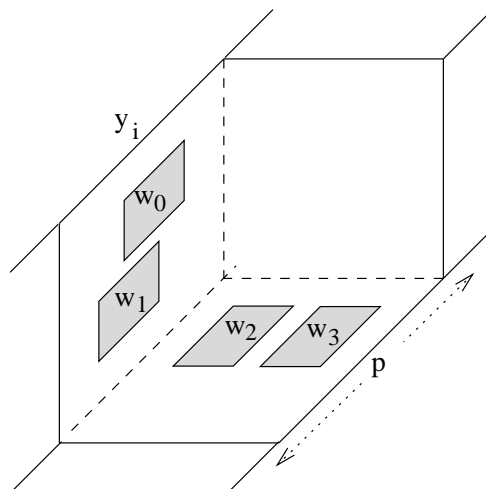


Fig. 4. An illustration of windows in a corridor

corridor metaphor. Figure 4 illustrates four windows, two located on the left surface and two located on the floor. The following are some of the ways in which windows can be configured.

- Geographic. Geographic information describes the physical placement of entities, including sensor locations as well as the interconnections between them. GPS, map or other coordinates can be used to specify exact locations. The locations can be projected onto a flat or topographic map. In keeping with common sense perception, topographic type windows should not be placed on the ceiling of the corridor.
- Trajectory Behavior. A trajectory of state evolution may tend towards states indicative of systems failure. While the existence of the degree of systems failure associated with a particular state along the trajectory is easily mapped to the visual elements of the corridor, details pertaining to the systems state are not so readily identifiable. Some aspects of interest include the following.
  - \* Sensor Correlation. Sensor data may be represented as a point in a flat 2D plot of  $\sqrt{M} \times \sqrt{M}$  window size where  $M$  is the number of sensors. In this representation, the depth of the window contains tuple represented data collectively de-

scribing each sensor’s details including its geographic location. A modifiable filter can be applied to eliminate uninteresting sensor information, for example, sensors which do not provide any information about the current state. A traversal window records the change of sensor related information from one state to the next.

- \* **State Semantics.** The linearization of the corridor disrupts the perception of the values and semantics of the state variables corresponding to the current state. The values and semantics of the associated state variables may be represented by projections of this state space onto the window. A modifiable filter can be applied to eliminate uninteresting information, for example, reducing out all dimensions which do not contribute to the current state. A traversal window records the change of state variables from one state to the next.
- **Region Behavior.** The state space may exhibit regions of behavior, that is, state transitions are describable by a single function applied to a region of the space. These behavior regions are indicative that small changes in certain state variables may locally have little change on the trajectory behavior. More global effects such as the tendency of the trajectory to evolve towards a systems failure may be influenced by the sequence of behavior regions through which the trajectory passes. From within a corridor, a behavior region is perceived to be external and surrounding the corridor. Some aspects of interest include the following.
  - \* **Region Flow.** Behavior regions indicate potential state changes both in the absolute state space and relative to the viewer. Perception of absolute behavior regions facilitates understanding about the global patterns involved in trajectories evolving towards system’s failure states. Relative perception how-

ever enables understanding about local changes and effects. A modifiable filter can be applied to eliminate uninteresting information, for example, removal of non-contributory dimensions thereby making the projected information more clear. A traversal window records the change of the region’s behavior from one state to the next.

### *C. Demonstration*

In this subsection, we demonstrate an immersive navigation prototype based on the visualization model described above. The current prototype is developed in 3Ds max. The purpose is to provide us initial feedback concerning the integration of the visualization model with the OOFSM data abstraction.

The prototype consists of an 18 second animation featuring a single corridor of five discrete states together with four traversal windows located on each of the walls, floor and ceiling. Color visualizations that are animated are displayed in each of the windows. Each state is visually separated by a color door frame. These colors are selected from a three color palette: green, white and red. Here, green signifies a transition into a nominal operating state, white signifies a transition into a state that potentially may lead to a systems failure, and red signifies a transition into a state that represents some system failure or cascading failure. The walls, floor and ceiling are not colored for data interpretation, but they are shaded to facilitate user perception of the corridor. Perspective views also facilitate user perception and enable depth cues. Simulation of both walking (forward motion) and head motion (changing viewing aspect) are included in the animation. Lastly, user interaction with one of the windows is also demonstrated in the animation.

Figure 5 shows an initial frame of the animation sequence. A pale-green door frame shows the transition from  $y_0$  to  $y_1$ , a white door frame shows the transition from  $y_1$  to  $y_2$ , the next white door frame, from  $y_2$  to  $y_3$ , a red door frame, from  $y_3$  to  $y_4$  and in the far distance, a white door frame, from  $y_4$  to  $y_5$ . In gray-scale, the door frames appear in very light gray, white and light gray, respectively, but all lighter as compared to the medium gray background

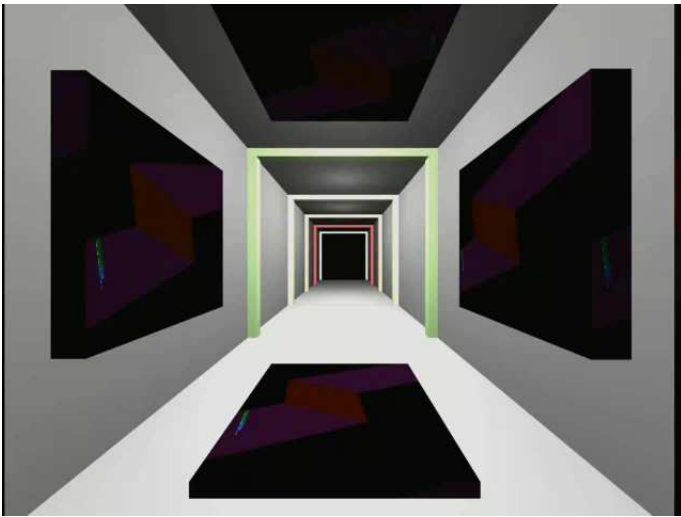


Fig. 5. Demonstration prototype: frame 1, state  $y_0$

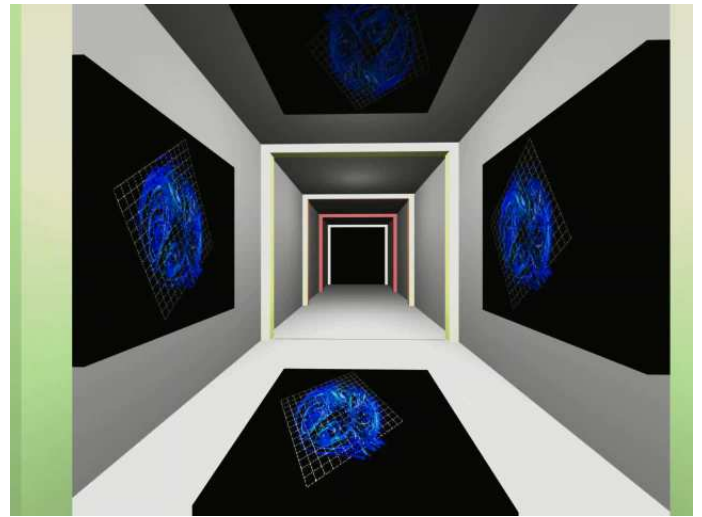


Fig. 7. Demonstration prototype: frame 3, state  $y_1$

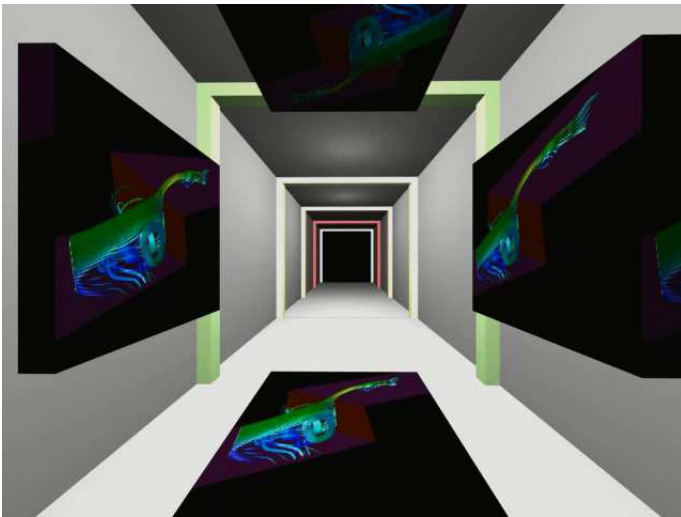


Fig. 6. Demonstration prototype: frame 2, state  $y_0$

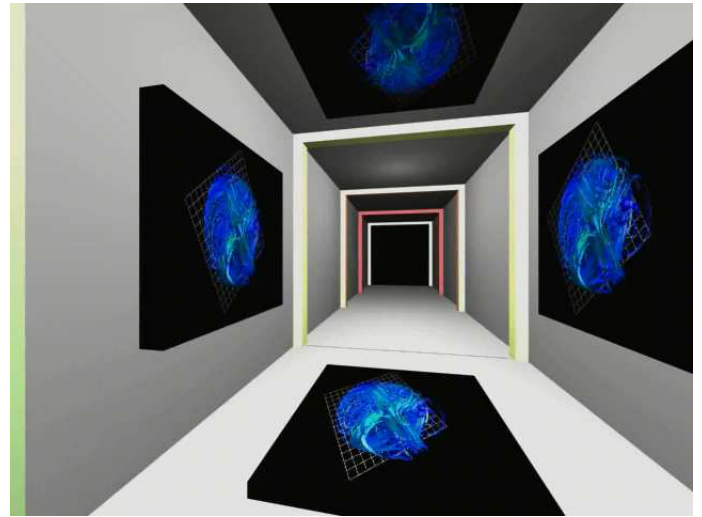


Fig. 8. Demonstration prototype: frame 4, state  $y_1$

of the walls and ceiling. There are four windows displayed in relief, one each on the walls, floor and ceiling. Most of the area of these windows are black in this frame.

Figure 6 shows a frame after the viewer has moved forward into state  $y_0$ . Here, the traversal nature of the windows becomes apparent as the windows maintain position relative to the viewer; note the partial occluding of the green (light gray) door frame by the relief of these windows. The windows themselves display a separate animated visualization that, in this demonstration, is time synchronized with the forward movement of the viewer through the corridor.

Figure 7 shows a frame just after traversing into state  $y_1$  as indicated by the pale green (light gray) door frame on the peripheral of the viewer. The animation in each of the windows has also changed.

Figure 8 shows a subsequent frame after the viewer has shifted the viewing aspect to focus more on the left window. Perspective of the corridor is maintained.

Figure 9 shows a user interaction with the left window. Here, the window is manipulated to be placed in front of the user. The user may then further interact with the window as an external reach-out-and-grab object. The corridor context is maintained throughout the interaction.

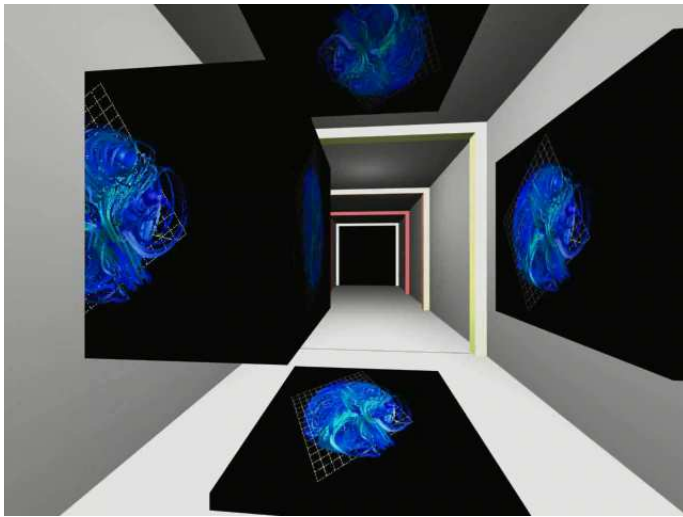


Fig. 9. Demonstration prototype: frame 5, state  $y_1$

So far, the demonstration has focused more on illustrating the visualization model. Now, we present somewhat more speculative comments regarding the appropriateness of the demonstration applied to the OOFSM exemplified by Figure 1. Let us consider  $y_0 = y_{1,2}$  and  $y_1 = y_{1,3}$ . For this simplified case, we ignore the remaining length of the corridor, the door frame colors, and the floor and ceiling windows. Figures 5 and 6 therefore pertain to OOFSM state  $y_{1,2}$  with  $\bar{V}_{R_2}$ . The left window would then display information about  $y_{0,2}$  and the right window, about  $y_{2,2}$  and/or  $y_{3,2}$ . Intuitively this information would be the corresponding direction vectors or region fields, but could be other types of information in general. The viewer therefore is immersed in a discrete trajectory represented by the corridor but is able to view the behavior of the surrounding state system. Figures 7, 8 and 9 pertain to state  $y_{1,3}$  with  $\bar{V}_{R_1}$ . The transition to this state and hence into a new region of behavior is supported by several features: the door frame and its color, possible textual tagging of  $\iota(b_{l_2(1,3)}) = 3$  to this door frame, and, the change of information displayed in the windows. The traversal nature of the windows facilitates the understanding of when, possibly how, the system's behavior changes with respect to the new state of the discrete trajectory. The manipulation of the window in this state is a useful operation in the event that the viewer wishes closer examination of the corresponding information.

### III. CONCLUSION

An immersive visualization model incorporating navigation, animation and other typical visualization features is proposed to facilitate the understanding of complex high-dimensional spatially-related data sets. The work is particularly motivated by understanding large-scale linear and non-linear controllable systems. However, it is not limited to these systems. The goals of the visualization model are to support the analysis of complex dynamic systems by analysts seeking to understand the operation and behavior of the system and at the same time to support the decision making process inherent in controlling the physical system. The visualization model is applied over a novel data structure termed Orthogonal Organized Finite State Machines (OOFSM) which defines a discrete abstraction of the underlying system. A brief review of OOFSMs is also included. A demonstration prototype has been developed and is described. It illustrates both the visualization model and its applicability to the OOFSM structure.

The demonstration prototype successfully shows the applicability of the proposed visualization model to the OOFSM. The feedback provided to us from the prototype helps us to consider improvements to our model. However, there remain many open questions including considering the issue of information display in the various windows and developing a working prototype visualization system.

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